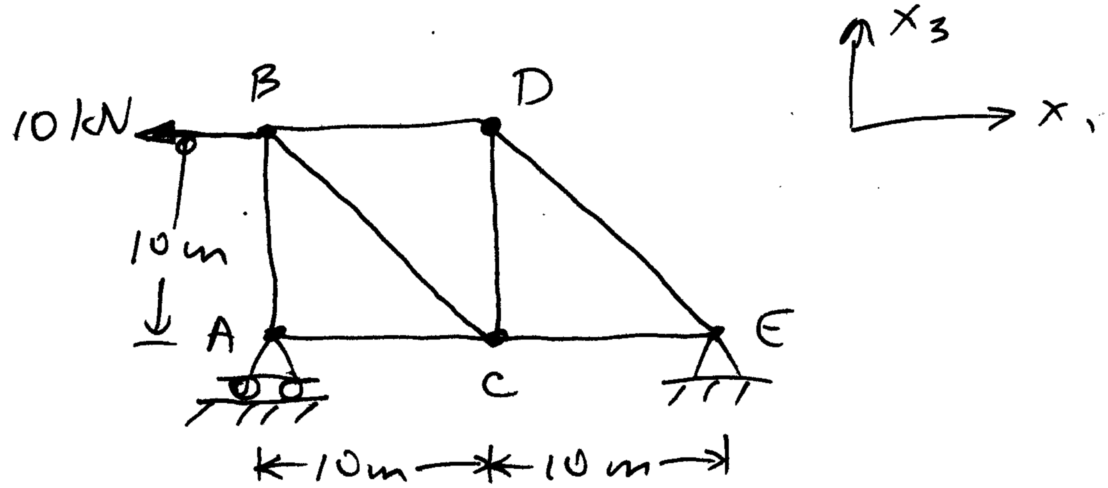
Pal
9/20/07

Unified Engineering Problem Set
Week 4 Fall, 2007

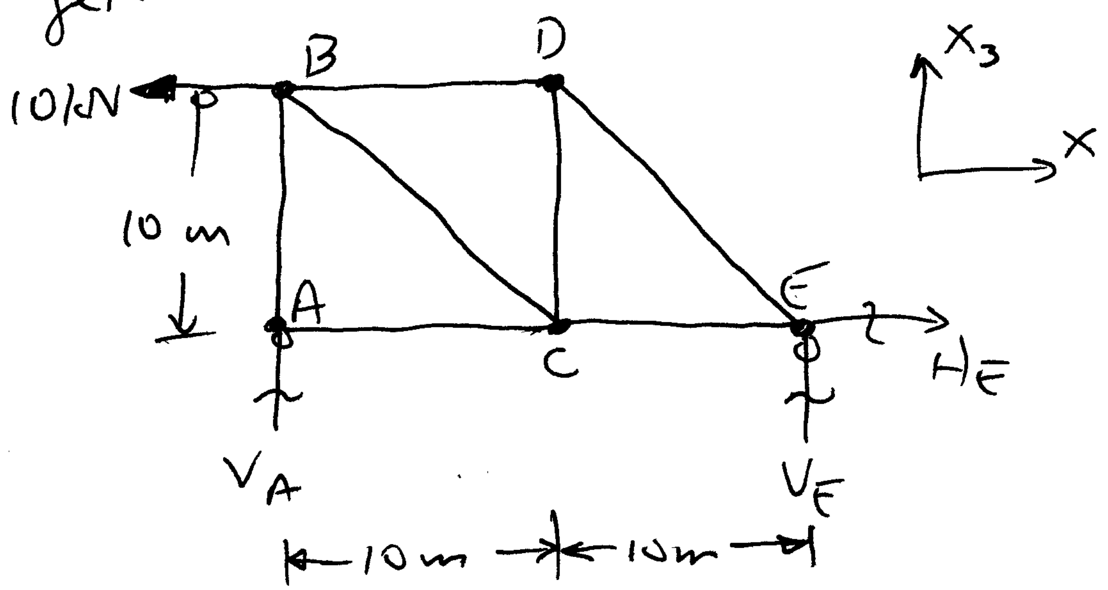
SOLUTIONS

M4.1

(a) Draw the Free Body Diagram of:



get:



(b) Use equilibrium.

$\sum F_1 = 0 \quad \xrightarrow{+} \quad \Rightarrow \quad -10kN + H_E = 0$

$\Rightarrow \boxed{H_E = 10kN}$

$\sum F_3 = 0 \quad \uparrow + \quad \Rightarrow \quad V_A + V_E = 0$

$\Rightarrow V_A = -V_E \qquad (1)$

$\sum M_A = 0 \quad \circlearrowleft + \quad \Rightarrow \quad (10kN)(10m) + V_E(20m) = 0$

take about point A (can use any point)

$\Rightarrow \boxed{V_E = -5kN}$

and from (1):

$\boxed{V_A = +5kN}$

Summarizing, the reactions are:

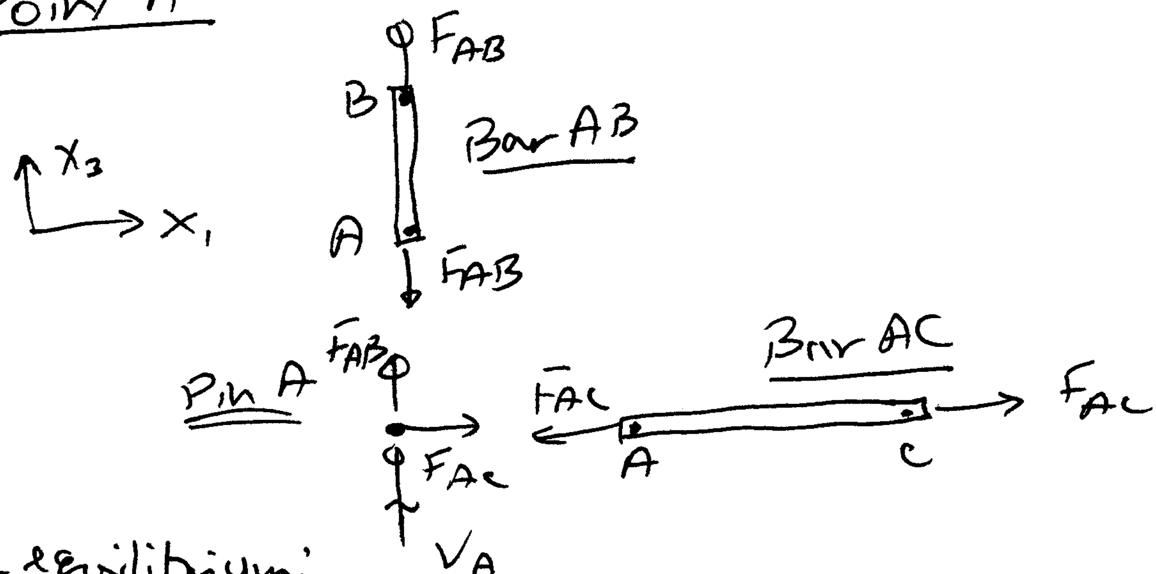
$$\boxed{\begin{aligned} H_E &= 10\,kN \\ V_E &= -5\,kN \\ V_A &= 5\,kN \end{aligned}}$$

(c) In working this, be consistent in the following:

- Put initial forces on bars in tension (away from bar)
- Bar forces acting on pins are equal and opposite to those in bar subsystem (summing them must result in zero)
- Reaction forces act on pins or determined
- Any bars at angles to axis system must have forces acting on pins resolved along axes

start with....

→ Point A



Use equilibrium:

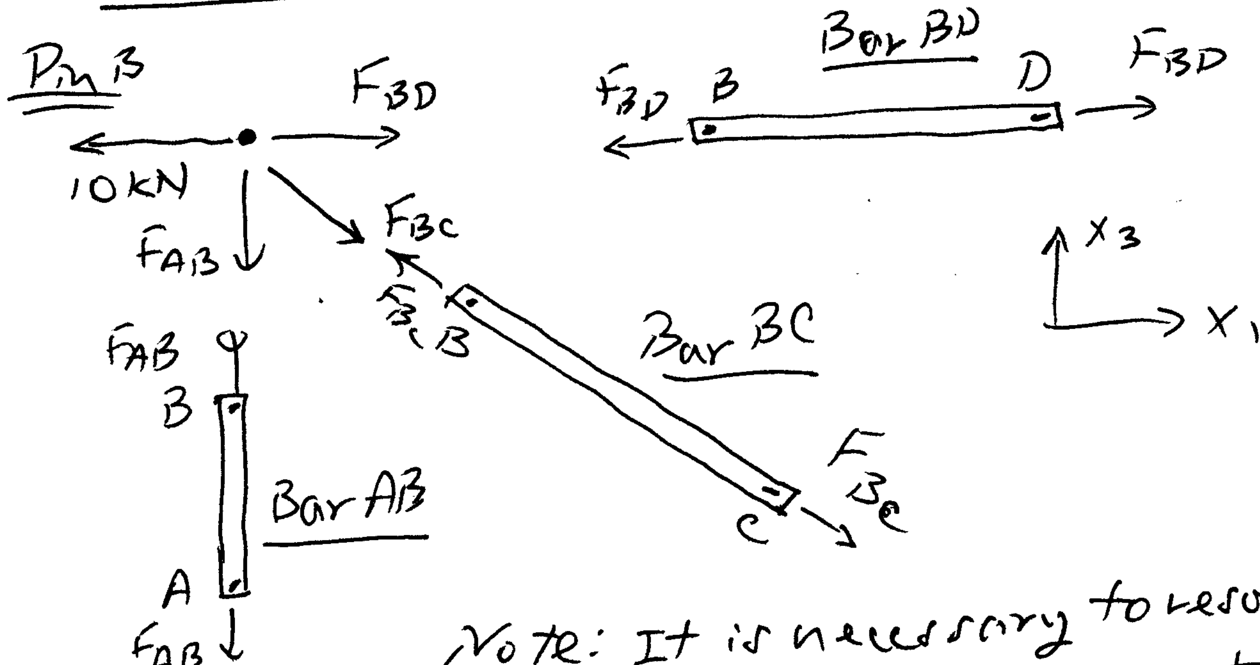
$\sum F_1 = 0 \ \overset{+}{\rightarrow} \Rightarrow \boxed{F_{AC} = 0}$

$\sum F_3 = 0 \ \uparrow + \Rightarrow \quad F_{AB} + V_A = 0$

$\Rightarrow \boxed{F_{AB} = -5kN}$

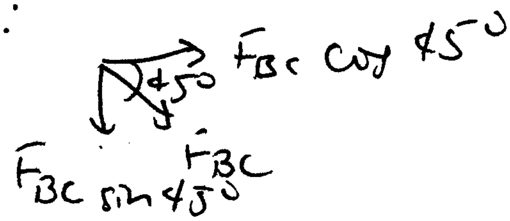
(NOTE: All forces act through pin so there are no moments to consider in this approach)

→ Point B



Note: It is necessary to resolve F_{BC} into x_1 and x_3 components. The length and height of the "bay" are equal (10 m), so the angle of the bar, and thus F_{BC}, is 45°.

So:



use equilibrium:

$\sum F_1 = 0 \overset{+}{\rightarrow} \Rightarrow -10kN + F_{BD} + 0.707 F_{BC} = 0$ (2)

$\sum F_3 = 0 \; \uparrow + \Rightarrow \; -F_{AB} - 0.707 F_{BC} = 0$

using value for $F_{AB} = -5kN$

$\Rightarrow \boxed{F_{BC} = 7.07kN}$

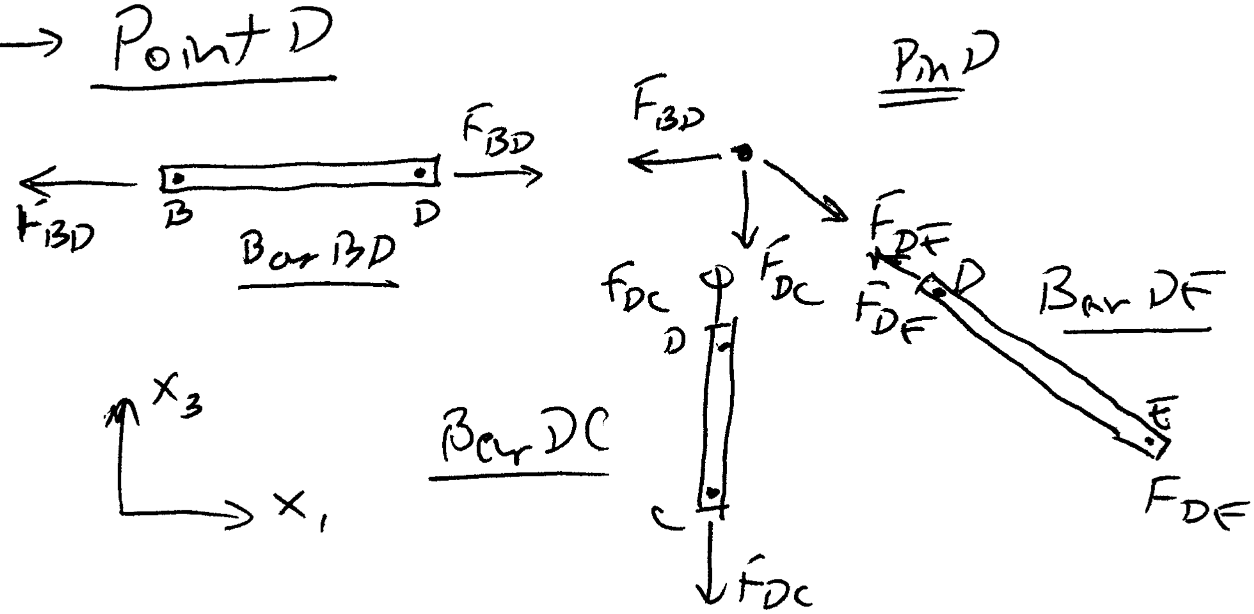
use that in (2):

$-10kN + F_{BD} + 5kN = 0$

$\Rightarrow \boxed{F_{BD} = 5kN}$

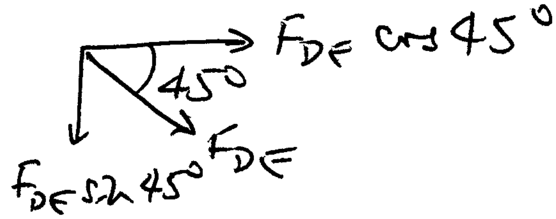
proceed to....

→ Point D



Note that it is again necessary to resolve F_{DF} into x_1 and x_3 components. This bar also has equal height and length of 10 m, so the angle is 45°.

This gives:



again using equilibrium:

$$\Sigma F_1 = 0 \quad \xrightarrow{+} \quad \Rightarrow \quad -F_{BD} + 0.707\, F_{DE} = 0$$

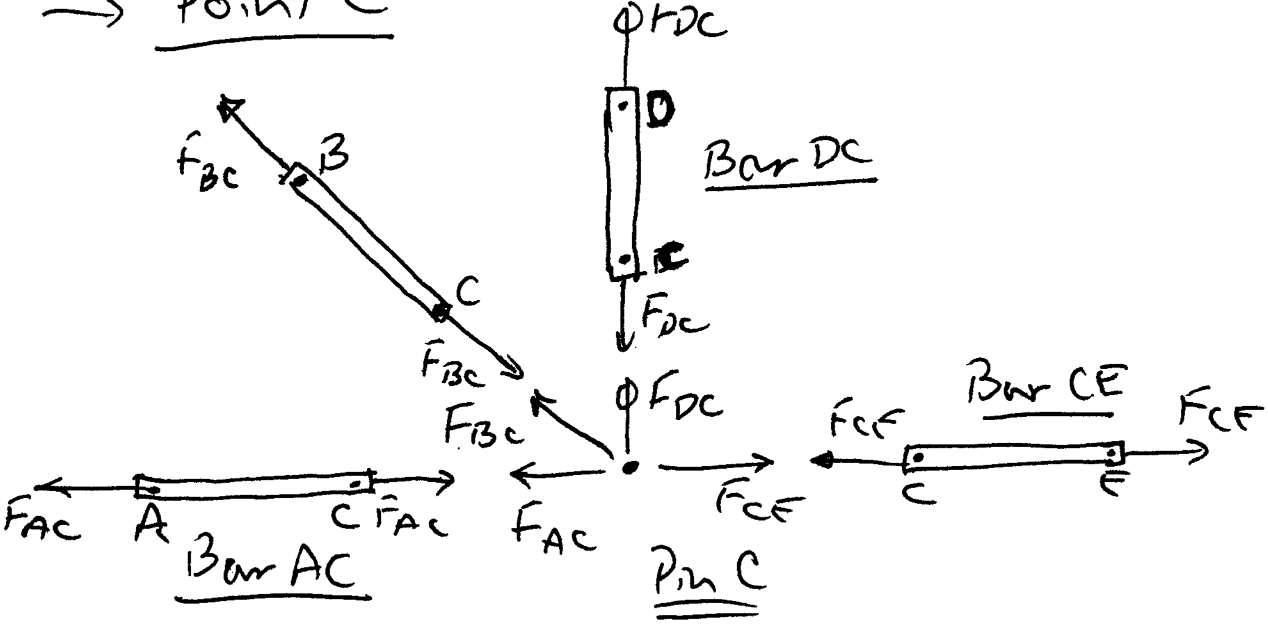
with $F_{BD} = 5kN \Rightarrow \boxed{F_{DE} = 7.07kN}$

$$\Sigma F_3 = 0 \quad \uparrow + \quad \Rightarrow \quad -F_{DC} - 0.707kN = 0$$

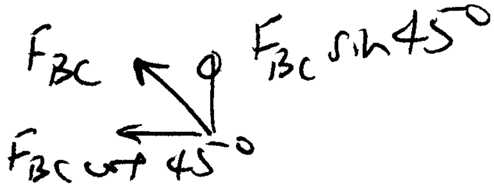
$$\Rightarrow \boxed{F_{DC} = -5kN}$$

now to ...

→ Point C



Again we know the bar BC is at an angle of 45°, so we resolve F_{BC} to be:



use equilibrium:

$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{AC} - 0.707 F_{BC} + F_{CE} = 0$

with $F_{AC} = 0$ and $F_{BC} = 7.07 kN$

$\Rightarrow \boxed{F_{CE} = 5 kN}$

$\sum F_3 = 0 \uparrow + \Rightarrow 0.707 F_{BC} + F_{DC} = 0$

have $F_{BC} = 7.07 N$ and $F_{DC} = -5 kN$

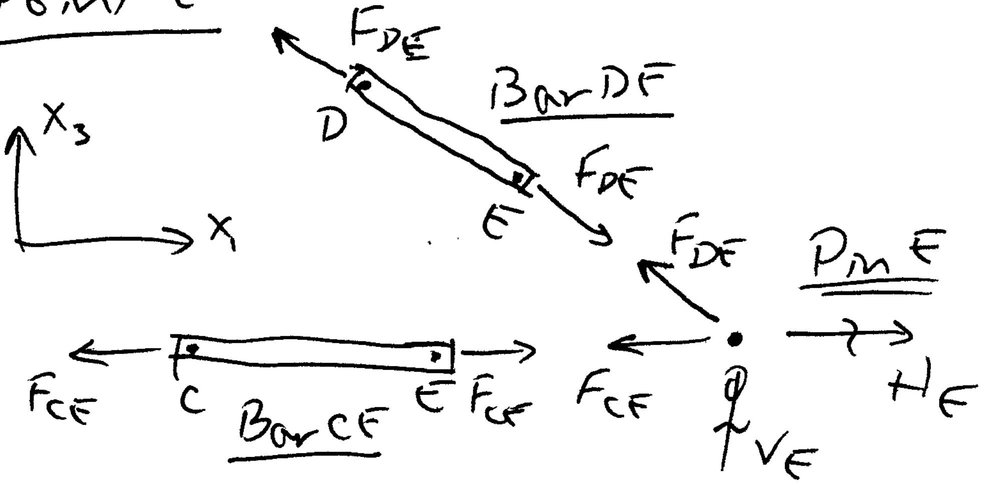
NOTE that there is redundancy in the application of equilibrium through the truss, so this serves as a check.

and $\quad 5kN - 5kN \stackrel{?}{=} 0 \quad$ YES

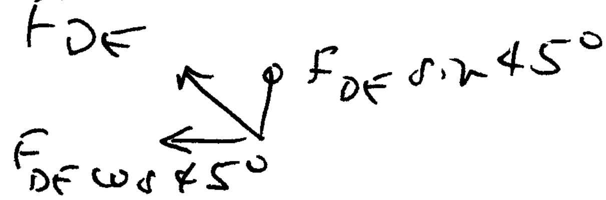
✓ checks

Finally get to...

→ Point E



Again, we know the bar DE is at an angle of 45°, so we resolve F_{DE} to be:



Also NOTE that we know all the loads acting at this point from our previous work, so the equilibrium applied here will serve as checks.

using that equilibrium....

$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow -F_{CE} - 0.707 F_{DE} + H_E = 0$

use the previously determined values of $F_{CE} = 5kN$, $F_{DE} = 7.07kN$, and $H_E = 10kN$

to give:

$-5kN - 5kN + 10kN \stackrel{?}{=} 0$ YES

✓ checks

$\Sigma F_3 = 0 \uparrow + \Rightarrow 0.707 F_{DE} + V_E = 0$

and adding that $V_E = -5kN$

gives:

$5kN - 5kN \stackrel{?}{=} 0$ YES

✓ checks

Summarize the results for the bar loads:

$$\boxed{\begin{aligned} F_{AC} &= 0 \\ F_{AB} &= -5\,kN \\ F_{BC} &= 7.07\,kN \\ F_{BD} &= 5\,kN \\ F_{DE} &= 7.07\,kN \\ F_{DC} &= -5\,kN \\ F_{CE} &= 5\,kN \end{aligned}}$$

Finally, draw the truss showing the applied load, reaction loads, and the loads of each bar: